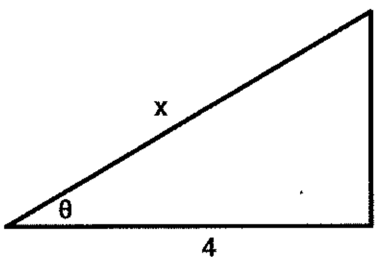
4.7 – Inverse Trig Functions

BELL WORK:

Use an inverse trigonometric function to write θ as a function of x

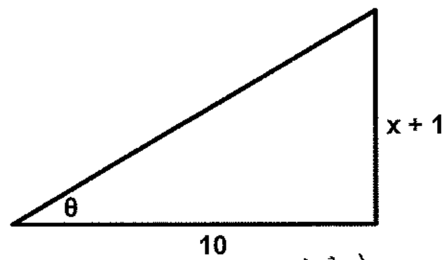
a)



$\cos\theta = \frac{4}{x}$

$\theta = \cos^{-1}\left(\frac{4}{x}\right)$

b)

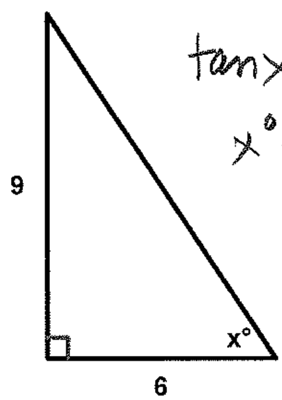


$\tan\theta = \frac{x+1}{10}$

$\theta = \tan^{-1}\left(\frac{x+1}{10}\right)$

REVIEW OF SOLVING A TRIANGLE

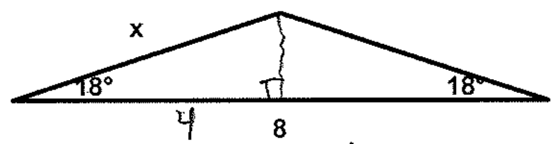
Example 1: Solve for the side or angle:



$\tan x^\circ = \frac{9}{6}$

$x^\circ = \tan^{-1}\left(\frac{3}{2}\right)$

$\approx 56.31^\circ$



$\cos 18^\circ = \frac{4}{x}$

$x = \frac{4}{\cos 18^\circ} \approx 4.21$ units

Example 2: Application

A security car with its spotlight on is parked 20 meters from a warehouse.
Consider θ and x as shown in the figure

a) Write θ as a function of x.

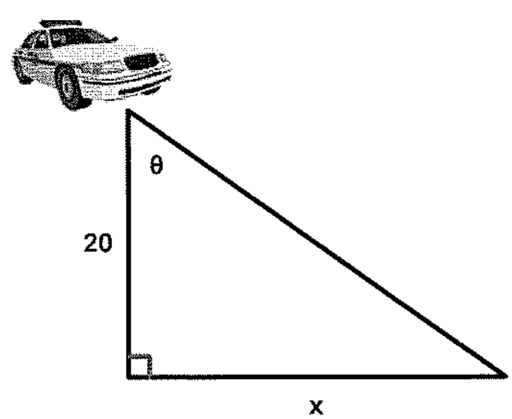
$\tan\theta = \frac{x}{20}$

$\theta = \tan^{-1}\left(\frac{x}{20}\right)$

b) Find θ when x=5 meters.

$\theta = \tan^{-1}\left(\frac{5}{20}\right)$

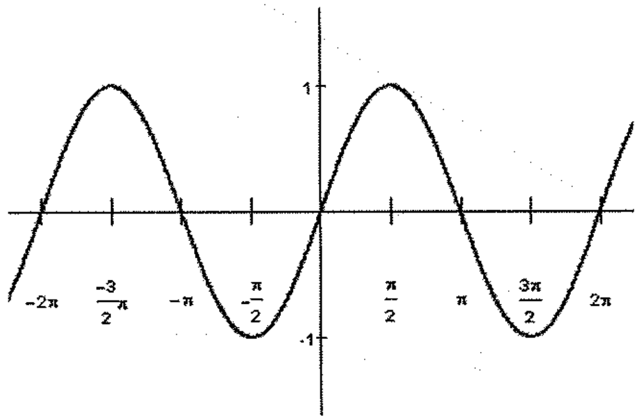
$\approx 14.04^\circ$



4.7 – Inverse Trig Functions

We have only ever explained inverse trigonometry in word problems with acute angles. What happens if the angle is NOT acute? First, we must consider what the sine function LOOKS like:

Sine Function



$y=\sin(x)$ IS a function

How do you know? passes the vertical line test!

The INVERSE of $y=\sin(x)$ is NOT a function

How do you know?

fails the horizontal line test

Even though the inverse of $y=\sin(x)$ is not a function, we study the inverse anyway because we need it to find angle measures! We look at a **small portion** of the graph (not all the x-values) that still hits all the y-values.

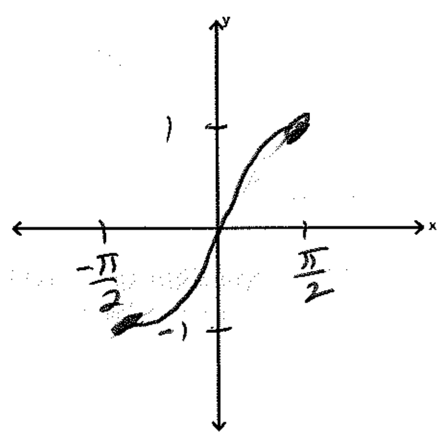
*******We restrict the domain to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ **********

On this interval...

1) y = sin(x) is increasing or decreasing? increasing

2) What is the domain? $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

3) What is the range? $[-1, 1]$



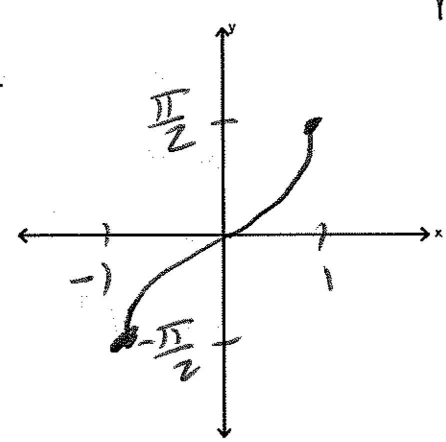
So when we graph the inverse function,

1) will it be increasing or decreasing? increasing

2) What is the domain? $[-1, 1]$

3) What is the range? $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

reflected over the line y=x



4.7 – Inverse Trig Functions

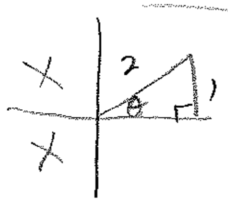
We use two types of notation for inverse functions:

y = $\sin^{-1}(x)$ or y = $\arcsin(x)$

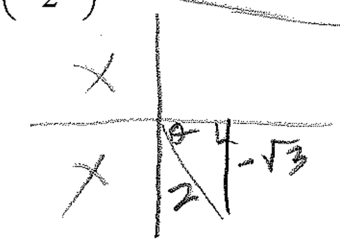
The idea behind the inverse problems is that they are GIVING you the value of the function, and you are trying to find the angle measure (in degrees or radians)

Example 1) Find the EXACT value (you are looking for ANGLE measures)
(hint: it may help to draw a right triangle)

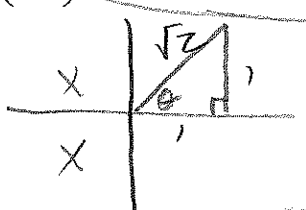
a) $\sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$ or $\frac{\pi}{6}$



b) $\arcsin\left(\frac{-\sqrt{3}}{2}\right) = -60$ or $-\frac{\pi}{3}$

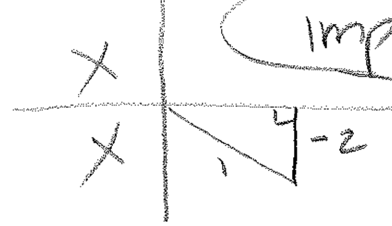


c) $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = 45^\circ$ or $\frac{\pi}{4}$

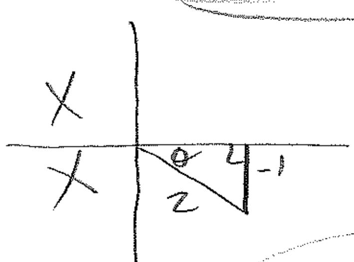


d) $\arcsin(-2) = \emptyset$

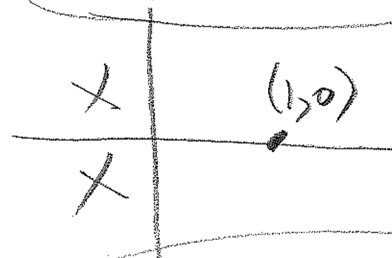
Impossible!!



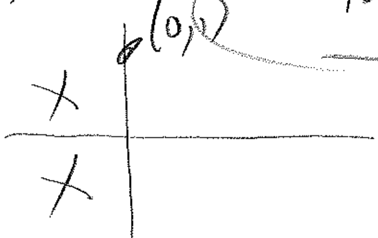
e) $\sin^{-1}\left(\frac{-1}{2}\right) = -30^\circ$ or $-\frac{\pi}{6}$



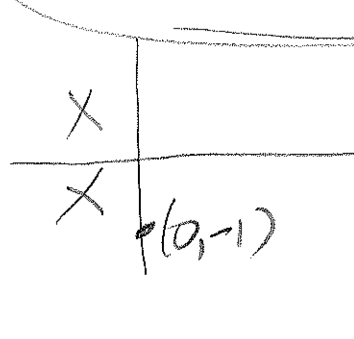
f) $\arcsin(0) = 0^\circ$ or 0π



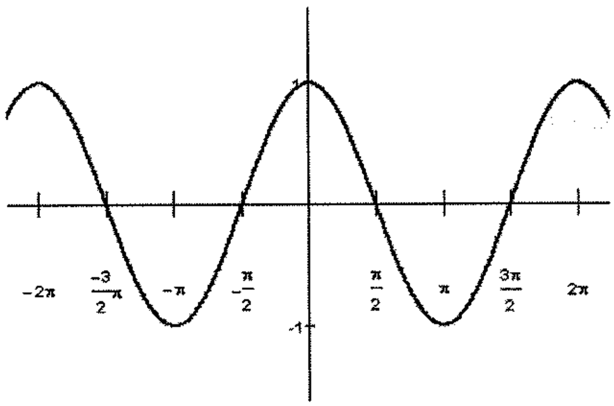
g) $\sin^{-1}(1) = 90^\circ$ or $\frac{\pi}{2}$



h) $\arcsin(-1) = -90^\circ$ or $-\frac{\pi}{2}$



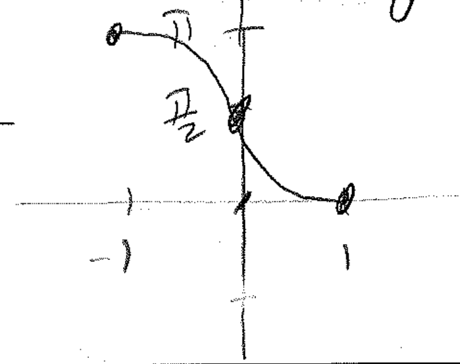
4.7 – Inverse Trig Functions



We restrict the Domain to: $[0, \pi]$

Range: $[-1, 1]$

Inverse graph:

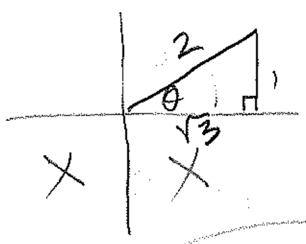


So for the inverse,

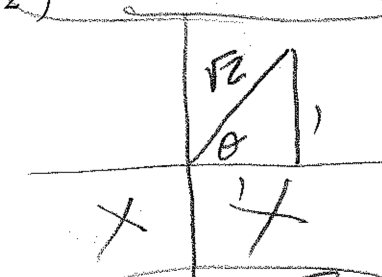
domain: $[-1, 1]$ range: $[0, \pi]$

Example 2) Find the exact value

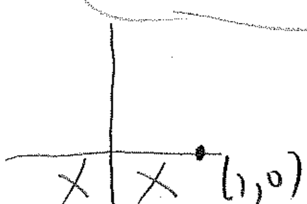
a) $\arccos\left(\frac{\sqrt{3}}{2}\right)$ $= 30°$ or $\frac{\pi}{6}$



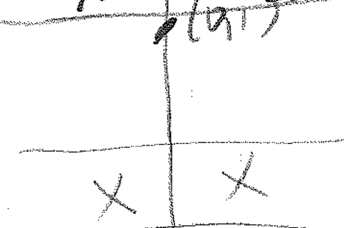
b) $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$ $= 45°$ or $\frac{\pi}{4}$



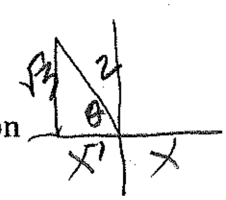
c) $\arccos(1)$ $= 0°$ or 0π



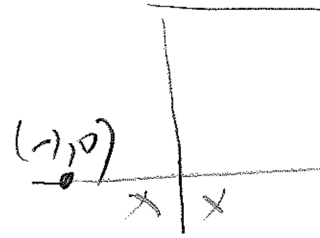
d) $\cos^{-1}(0)$ $= 90°$ or $\frac{\pi}{2}$



e) $\cos^{-1}\left(\frac{-1}{2}\right)$ $= 120°$ or $\frac{2\pi}{3}$

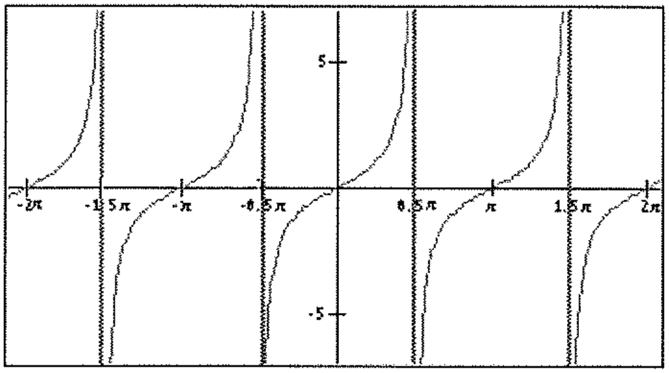


f) $\arccos(-1)$ $= 180°$ or π



Tangent Function

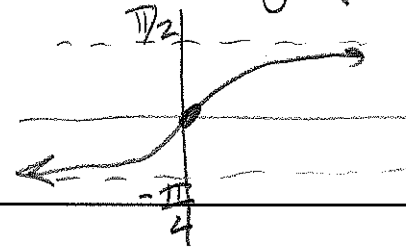
4.7 – Inverse Trig Functions



We restrict the domain to: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Range: $(-\infty, \infty)$

Inverse graph:

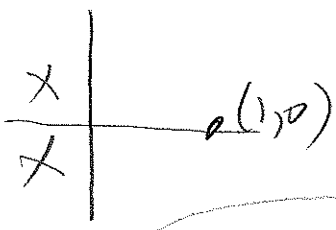


So for the inverse,

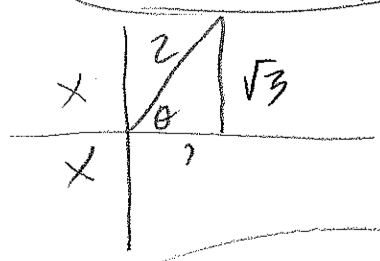
domain: $(-\infty, \infty)$ range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Example 3) Find the exact value

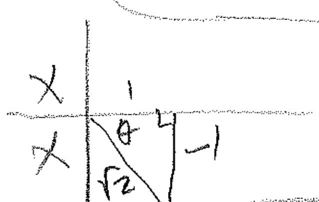
a) arctan(0) $= 0°$ or 0π



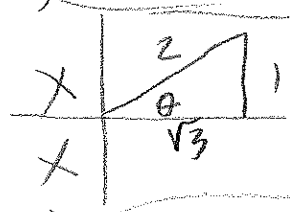
b) $\tan^{-1}(\sqrt{3})$ $= 60°$ or $\frac{\pi}{3}$



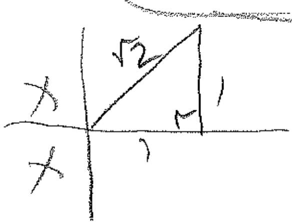
c) arctan(−1) $= -45°$ or $-\frac{\pi}{4}$



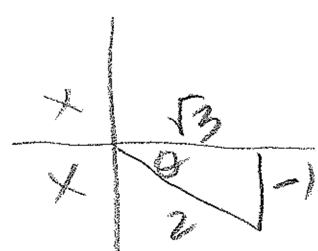
d) $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$ $= 30°$ or $\frac{\pi}{6}$



e) arctan(1) $= 45$ or $\frac{\pi}{4}$



f) $\tan^{-1}\left(\frac{-\sqrt{3}}{3}\right)$ $= -30°$ or $-\frac{\pi}{6}$



Homework for Day 1: p.349 # 1-16